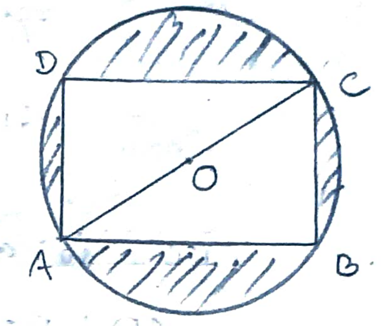
Exercise : 12.3.

1. Here, $AB = CD = 8$ cm

$AD = BC = 6$ cm



$\therefore AC = \sqrt{8^2+6^2} = 10$ cm.

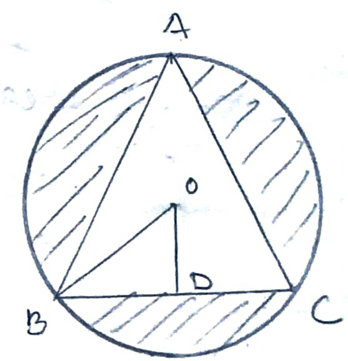
$\therefore AO$ = Radius of the circle = 5 cm.

Now, area enclosed between the circle and the rectangle $= \pi r^2 - \cancel{6\times8}$.

$= \frac{22}{7} \times 25 - 48 = \frac{550}{7} - 48$.

$= \frac{550 - 330}{7} = 30.57$ cm^2

2. Here, $OD \perp BC$, OB = radius



$\therefore BD = OB \times \frac{BD}{OB} = 14 \times \cos 30°$

$= 14 \times \frac{\sqrt{3}}{2} = 7\sqrt{3}$ cm.

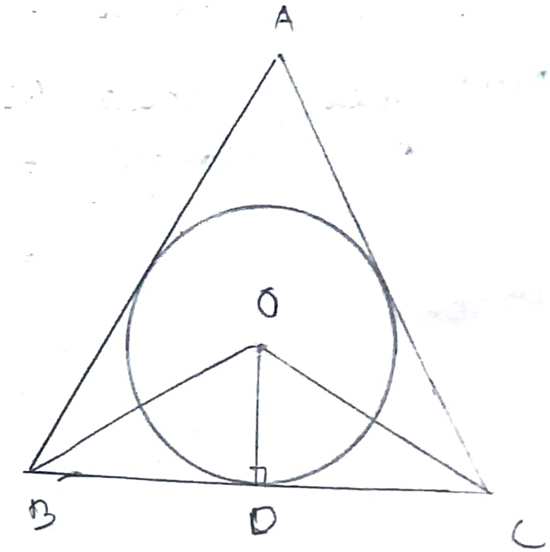
$\therefore BC = 2BD = 14\sqrt{3}$ cm.

Now, area of shaded portion = area of circle − area of equilateral $\triangle$

$= \frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}} \times 14 - \frac{\sqrt{3}}{4} \times (14\sqrt{3})^2$

$= 616 - 259.31 \quad = 361.69 \text{ cm}^2.$

3. $\triangle ABC$ is an equilateral triangle.



$\therefore$ $OD \perp BC$ and OD bisects BC at D

$\therefore$ $BD = DC = 9$ cm.

$\frac{OD}{BD} = \tan 30^\circ = \frac{1}{\sqrt{3}}.$

$\Rightarrow \frac{OD}{9} = \frac{1}{\sqrt{3}}$

$\Rightarrow OD = \frac{9}{\sqrt{3}} = 3\sqrt{3}$ cm.

$\therefore$ area between the triangle and the circle

$= \frac{\sqrt{3}}{4} \times 18^2 - \pi (3\sqrt{3})^2 \text{ cm}^2.$

$= \frac{1.73}{\cancel{4}} \times \overset{9}{\cancel{18}} \times \overset{9}{\cancel{18}} - \frac{22}{7} \times 27 \text{ cm}^2.$

$= 140.13 - 84.85 \quad = 55.28 \text{ cm}^2$ (approx)

4. From the fig,

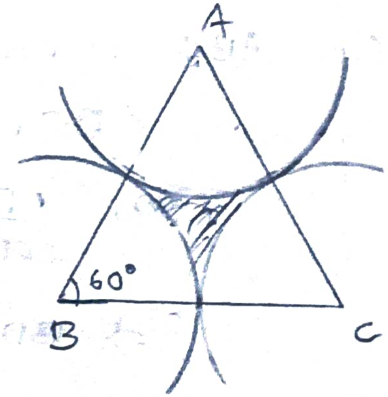
Area of the shaded region

$= \text{area of } \triangle ABC - 3(\text{area of sector})$

$= \frac{\sqrt{3}}{4} \times 12^2 - 3 \times \frac{22}{7} \times 6 \times \cancel{6} \times \frac{\cancel{60}}{\cancel{360}_{\cancel{6}}}$

$= \frac{1.873}{4} \times 144 - \frac{66}{7} \times 6$

$= 62.28 - 56.57 \quad = 5.71 \text{ cm}^2 \text{ (approx)}$



5. Here, $AB = BC = CD = DA = 56 \text{ m}$.

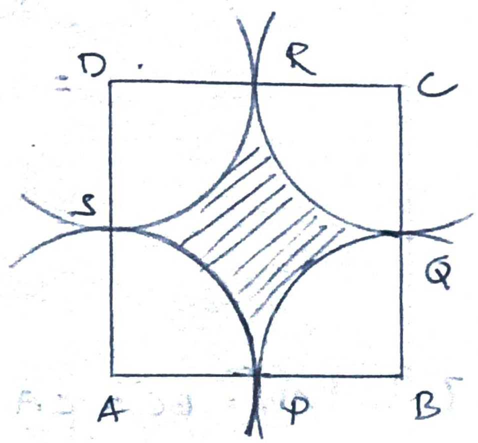
$\therefore \; AP = 28 \text{ m}$.

Now, area of shaded portion

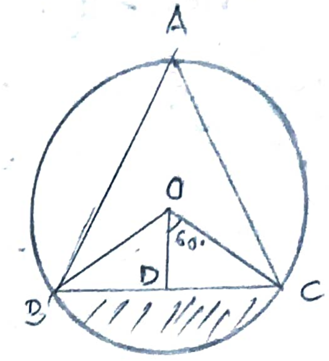
$= \text{area of square } ABCD - 4(\text{area of sector } \widehat{APS})$

$= 3136 - 4 \times \frac{22 \times 28 \times \cancel{28} \times \cancel{90}}{\cancel{7} \times \cancel{360}_{\cancel{4}}}$

$= 3136 - 2464 \quad = 672 \text{ m}^2$



6. ABC is an equilateral triangle



$\therefore \angle BOD = 60°$.

Thus, $\frac{BD}{\cancel{OB}\,OD} = \sin 60° = \frac{\sqrt{3}}{2}$.

$\Rightarrow BD = \frac{7 \times 1.73}{2} = 6.055$ cm.

$OD = \frac{7}{2} = 3.5$ cm.

$\therefore BC = 2 \times 6.055 = 12.11$ cm.

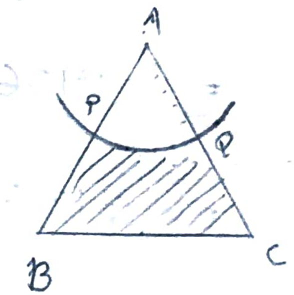
$\therefore$ Area of minor segment = area of sector OBC − area of $\triangle BOC$.

$$= \frac{\cancel{120}}{\cancel{360}_3} \times \frac{22}{7} \times 7 \times 7 - \frac{1}{2} \times 12.11 \times 3.5$$

$$= \frac{1078}{21} - \frac{42.385}{2} = 51.33 - 21.19$$

$$= 30.14 \text{ cm}^2.$$

7 $AB = BC = CA = 14$ cm.



$AP = AQ = 7$ cm.

$\angle A = 60°$.

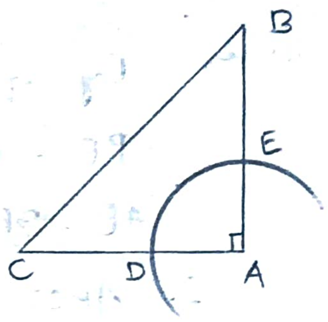
Now, area of shaded region
= area of equilateral triangle − area of sector

$= \frac{\sqrt{3}}{\cancel{4}} \times \overset{7}{\cancel{14}} \times \overset{7}{\cancel{14}} - \frac{\overset{11}{\cancel{22}} \times 7 \times \cancel{7} \times \cancel{60}}{\cancel{7} \times \cancel{360}_{6_3}}$

$= 1.73 \times 49 - \frac{77}{3}$

$= 59.13 cm^2$.

8. Area of the remaining portion



$= $ Area of triangle $-$ area of sector.

$= \frac{1}{\cancel{2}} \times \overset{4}{\cancel{8}} \times 8 - \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \overset{0.5}{\cancel{3.5}} \times 3.5 \times \frac{\cancel{90}}{\cancel{360}_{\cancel{4}_2}}$

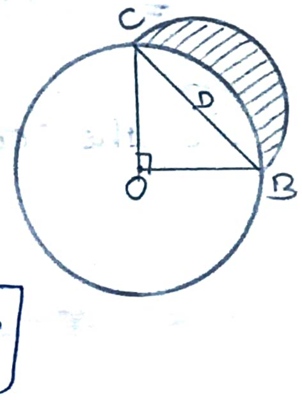
$= 32 - 9.625$.

$= 22.38 cm^2$ (approx)

9. Here, $OB = OC = 8 cm$.

$\therefore BC = \sqrt{8^2+8^2} = 8\sqrt{2} cm$.

$\therefore$ radius $= 4\sqrt{2} cm = BD$



Now, area of shaded region

$=$ Area of semi-circle $-$ area of minor segment

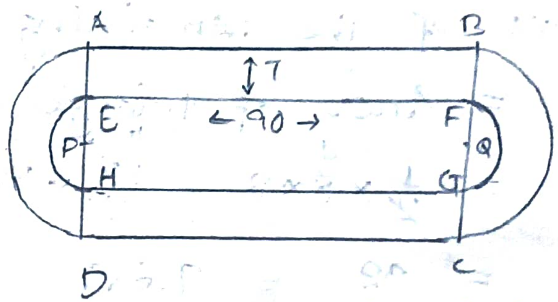
$= \frac{1}{\cancel{2}} \times \frac{\overset{11}{\cancel{22}}}{7} \times (4\sqrt{2})^2 - \frac{8^2}{2}\left[\frac{22 \times 90}{7 \times 180} - \sin 90°\right]$

$$= \frac{352}{7} - 32 \times \frac{4}{7} = \frac{224}{7}$$

$$= 32 \text{ cm}^2.$$

10\. Here,

By question, we have



$PE = PH = 35$ cm.

$AE = BF = CG = HD = 7$m

$\therefore$ Area of the track

= Ar. of rectangle ABEF + ar. of rectangle CDHG + 2(ar. of semi circle)

$= (90 \times 7) + (90 \times 7) + 2\left\{\frac{\pi}{2}(R^2 - r^2)\right\}$

$= 1260 + \frac{22}{7} \times (42+35)(42-35).$

$= 1260 + 1694 = 2954 \text{ cm}^2.$

$\therefore$ the cost of turfing the track = ₹ (2954×250)

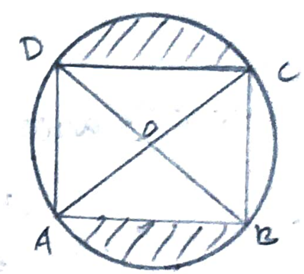
= ₹ 7,38,500.

11 $AB = BC = CD = DA = 42\,m.$

$\angle AOB = 90^\circ$ and $\angle ABC = 90^\circ$.

In right $\triangle ABC$, $AC = \sqrt{42^2+42^2} = 42\sqrt{2}\,m.$

$\therefore$ Radius $= 21\sqrt{2}\,m.$



Now, area of two beds

$= 2 \times$ area of minor segment $\widehat{AB}$

$$= 2 \times \frac{(21\sqrt{2})^2}{2}\left[\frac{22\times 90}{7\times 180} - 1\right]$$

$$= 441 \times 2 \times \frac{4}{7} = 504\,m^2.$$

Thus, the cost of planting $=$ ₹ 504×200

$=$ ₹ 1,00,800

12 Here, area of rectangular window $= 84 \times 56 = 4704\,cm^2$

total area of 24 circular design $= 24 \times \frac{22}{7} \times 7 \times 7$

$= 3696\,m^2$

$\therefore$ cost of making designs $= \frac{3696 \times 25}{20} =$ ₹ 4620.

and area of the remaining portion $= 4704 - 3696$

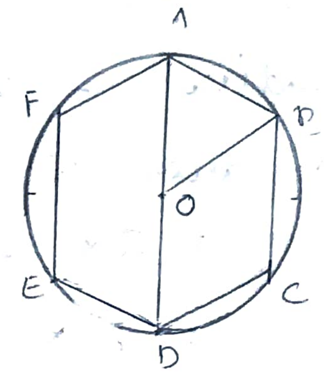
$= 1008\ cm^2$.

13. Here, area of the hexagon

$= 6 \times$ area of ΔAOB.

$= 6 \times \frac{1.73}{4} \times 56 \times 56$

$= 8137.92\ cm^2$



Then, area of the circle $= \frac{22}{7} \times 56 \times 56$

$= 9856\ cm^2$.

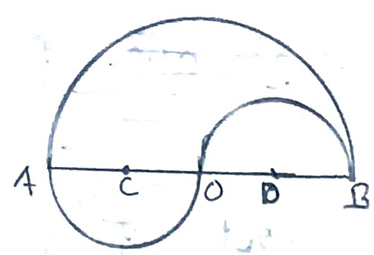
$\therefore$ area of designing portion $= 9856 - 8137.92$.

$= 1718.08\ cm^2$.

Thus, cost of designing $= \frac{1718.08 \times 8}{15} = ₹\ 916.30$.

14. Here, $OA = OB = 28\ cm$.

$AC = OC = OD = DB = \frac{1}{2}\ 40 = 14\ cm$.



Now, area of shaded region

= ar. of semicircle with centre O + ar. of semi-circle with ~~radius~~ centre C − ar. of semi circle with centre D.

= area of semi circle with radius AO.

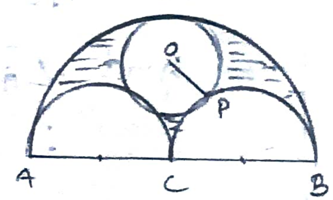
$= \frac{1}{\cancel{2}} \times \frac{\cancel{22}^{11}}{\cancel{7}} \times \cancel{28}^{4} \times 28 \quad = 1232 \text{ cm}^2.$

15. Here,

AC = BC = 18 cm.

∴ AB = 36 cm.

$OP = \frac{1}{6} AB = 6 \text{ cm}.$



Thus, area of semi circle with radius 18 cm,

$= \frac{1}{2}\pi r^2 \quad = \frac{1}{2} \times \frac{22}{7} \times 18 \times 18 \text{ cm}^2$

and area of circle with radius OP $= \frac{22}{7} \times 6 \times 6 \text{ cm}^2$

area of semi-circles with radius 9 cm $= \frac{1}{2} \times \frac{22 \times 9 \times 9}{7} \times 2$

Now, area of shaded portion

$$= \frac{22 \times 18 \times 18}{7} - \left(\frac{1}{2} \times \frac{22 \times 9 \times 9}{7} \times 2 + \frac{22}{7} \times 6 \times 6\right)$$

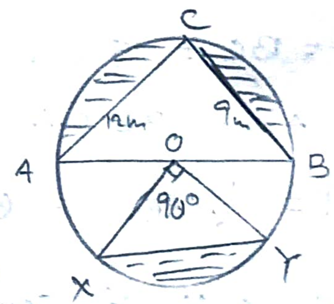
$$= 509.14 - (254.57 + 113.19)$$

$$= 141.43 \text{ cm}^2.$$

16. Here, $AC = 12$ m. $BC = 9$ m

and $\angle ACB$ is in a semi-circle

$\therefore \angle ACB = 90°$.



Then, using Pythagoras theorem, we get

$AB = 15$.

Thus, $OA = OB = \frac{15}{2}$ m. $= 7.5$ m.

Area of semi-circle (ACB) $= \frac{1}{2}\pi r^2 = \frac{1 \times 22 \times 7.5 \times 7.5}{2 \times 7}$

$= 88.39 \text{ m}^2$.

Area of $\Delta ABC = \frac{1}{\cancel{2}} \times \overset{6}{\cancel{12}} \times 9 = 54 \text{ m}^2$.

and area of minor segment XPY $= \frac{r^2}{2}\left[\frac{\pi\theta}{180} - \sin\theta\right]$

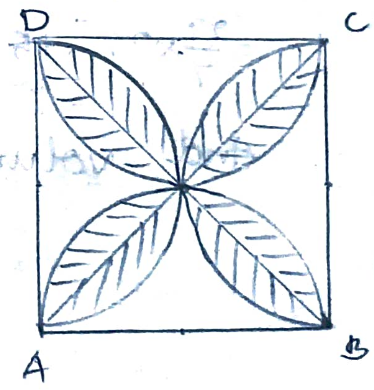
$= \frac{56.25}{2}\left[\frac{90\times22}{360\times7} - 1\right]$

$= 16.07.$

$\therefore$ Reqd. area $= 88.39 + 16.07 - 59\ m^2$

$= 50.46\ m^2.$

17. Here,



$AB = BC = CD = DA = 7cm.$

$\therefore$ radius of semi circle $= 3.5\ cm.$

Thus, area of petals

$= 8 \times$ area of a segment.

$= \cancel{8}^{4} \times \frac{3.5\times3.5}{\cancel{2}}\left[\frac{\cancel{22}^{11}\times 90}{7\times\cancel{180}_{2}} - 1\right]$

$= \cancel{49}^{7} \times \frac{4}{\cancel{7}} = 28\ cm^2$

$\therefore$ cost of design $= ₹\,(12\times28)$

$= ₹\ 336.$